

# Bound states in the 3d Ising model and implications for QCD at finite temperature and density

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We study the spectrum of bound states of the three dimensional Ising model in the  $(h, \beta)$  plane near the critical point. We show the existence of an unbinding line, defined as the boundary of the region where bound states exist. Numerical evidence suggests that this line coincides with the  $\beta = \beta_c$  axis. When the 3D Ising model is considered as an effective description of hot QCD at finite density, we conjecture the correspondence between the unbinding line and the line that separates the quark-gluon plasma phase from the superconducting phase. The bound states of the Ising model are conjectured to correspond to the diquarks of the latter phase of QCD.

## 1. Introduction

The use of effective models to describe the relevant degrees of freedom has always been an important tool in the study of non perturbative physics of gauge theories. For pure gauge theories with a second order deconfinement transition, the problem of finding such an effective description is solved by the Svetitsky-Yaffe conjecture [1].

The case of finite density QCD is not as well understood. It is by now clear that in the phase diagram of high  $T$  and finite density QCD a central role is played by the second order phase transition located at the end of the first order line which separates the hadronic phase from the quark-gluon plasma one at high density (see fig.1). It has been conjectured that this point is in the same universality class as the order-disorder transition of the 3d Ising model. This identification is even more important than the Svetitsky-Yaffe one for the zero density case, since the difficulty of directly simulating finite density QCD makes an effective description in terms of Ising-type variables more valuable.

The critical behaviour of the 3d Ising model is by now very well understood both numerically and field-theoretically. In particular we have a

good understanding of the properties of the free energy and its derivatives near the critical point.

However, in view of an application to finite density QCD one is more interested in phenomena (like the Kertész singularity [2] or the unbinding line which we shall discuss below) which are *not* related to any singularity in the free energy.

The goal of this contribution is to discuss these quantities, their behaviour in the 3d Ising model and the possible implications for finite density QCD. Here we just state our results: for all details and derivations, see [3,4].

## 2. Bound states in the 3d Ising model

The 3D Ising model in the broken symmetry phase has a rich spectrum of non perturbative states, which can be interpreted as bound states of the fundamental massive excitation [3].

This spectrum is mapped by duality into the glueball spectrum of the gauge Ising model, and hence includes states with all the values of angular momentum  $J$  and parity  $P$  allowed by the lattice structure. However for our current purposes we can focus on the simplest one, that is the  $J^P = 0^+$  bound state of two elementary quanta.

At vanishing magnetic field, numerical simula-

tions of the Ising model show the existence of a state with mass  $\sim 1.83m$ , where  $m$  is the mass of the fundamental excitation (inverse of the correlation length). On the other hand, the Bethe-Salpeter equation for  $3D \phi^4$  theory in the broken symmetry phase predicts the existence of a bound state of two fundamental quanta, with binding energy given at leading order by

$$E_b \sim m \exp\left(-\frac{8\pi m}{g}\right) \sim 0.17m \quad (1)$$

The excellent agreement between these numbers allows us to interpret the state at  $\sim 1.83m$  as a bound state.

The Bethe-Salpeter equation shows also that when an external magnetic field is switched on the binding energy decreases. Moreover, in the unbroken symmetry phase the interaction between fundamental excitations is repulsive, and no bound states can exist. These two facts suggest the existence of an *unbinding line* in the  $(T, H)$  plane delimiting the region where bound states exist. On the line the binding energy vanishes. High precision Monte Carlo data confirm the existence of this line and strongly suggest that it coincides with the  $T = T_c$  axis.

### 3. “Critical” and pseudocritical lines in the $(H, T)$ plane.

The unbinding line  $E_b(H, T) = 0$  shares many features with the *Kertész line*, along which the cluster surface tension  $\Gamma(H, T)$  vanishes (see [5] for the precise definition). Given a scaling function  $X(h, t)$ , expressed in terms of the reduced variables  $h \equiv \beta H$  and  $t = \frac{T - T_c}{T_c} \equiv \frac{\beta_c - \beta}{\beta}$ , a simple renormalization group argument shows that the line along which  $X$  vanishes must have the form

$$t = a_X |h|^{\frac{y_t}{y_h}} \quad (2)$$

where  $y_t$  and  $y_h$  are the RG eigenvalues of the energy and spin operators. The constant  $a_X$  depends on the choice of the observable  $X$ .

Since both  $E_b$  and  $\Gamma$  are scaling functions, both the unbinding line and the Kertész line will be described by Eq. 2. Numerically we have for the unbinding line

$$a_{E_b} \sim 0 \quad (3)$$

while for the Kertész line [5]

$$a_\Gamma \sim 0.39 \quad (4)$$

This indicates that the two lines are definitely different. Both can be considered as “critical lines” since they divide the  $(h, t)$  plane into well defined phases which can be distinguished by an order parameter ( $E_b$  or  $\Gamma$ ) in the two cases, which is different from zero in one phase and vanishes in the other phase. However these are not phase transitions in the usual sense since the free energy is not singular on either of these lines.

It is interesting to compare the Kertész and unbinding line to the so called *pseudocritical* lines, defined as the loci of the maxima in the  $(H, T)$  plane of quantities, like the susceptibility  $\chi(h, t)$ , the specific heat  $C(h, t)$  or the correlation length  $\xi(h, t)$  which diverge at the critical point. Since also these quantities are scaling functions, the functional form of the pseudocritical lines must be the same of eq.(2), with various constants  $a_\chi, a_C, a_\xi, \dots$

It is very interesting to compare the various values of these constants. It seems that the pseudocritical lines divide into two families, which are, so to speak, “attracted” by the Kertész and unbinding lines respectively. In particular it turns out that  $a_\chi \sim a_\xi \sim a_\Gamma$  and  $a_C \sim a_{E_b}$ . While the susceptibility pseudocritical line lies near the Kertész line but is definitely different from it (a similar behaviour was recently observed also in  $d=2$  [6]) the pseudocritical line related to the correlation length seems to almost coincide with the Kertész line. It would be very interesting to understand if this is only a coincidence, if a higher resolution analysis separates the two lines and if a similar phenomenon also holds in other models or in the  $2d$  Ising model.

### 4. Implications for finite density QCD.

Our current understanding of finite  $\mu$  and  $T$  QCD is well summarized by fig. 1 (taken from [7]).

In particular, as  $\mu$  increases we recognize three regimes: (1) At low  $\mu$ , since the chiral symmetry is not exact the hadronic phase is separated from the quark-gluon plasma (QGP) one by a

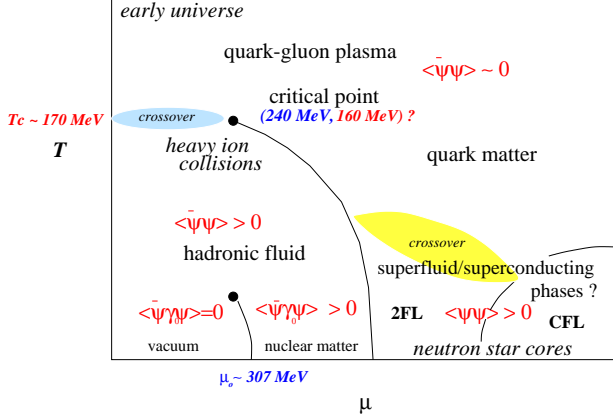


Figure 1. Schematic view of the QCD phase diagram.

smooth crossover. (2) At intermediate values of  $\mu$  the two phases are separated by a first order line whose ending point belongs to the Ising universality class. (3) At large  $\mu$  a new “superconducting” (SC) phase appears in which quarks pair and can form a condensate.

Again the SC phase is separated by the QGP only by a smooth crossover since both are characterized by the same global symmetries. The boundary between the two is only defined by the fact that the binding energy between quarks, which is [8]

$$E_b \sim \frac{b}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) \quad (5)$$

becomes zero.

If the endpoint of the line of first order transition is in the Ising universality class, then all critical indices and universal amplitude ratios coincide with the Ising ones. It is natural to wonder whether this identification provides us with useful insight about the physically interesting crossover phenomena around the critical point. In this spirit, it was conjectured in [9] that the Kertész line is to be identified with the separation between hadronic and QGP phase. Here we propose to identify the unbinding line with the line separating the SC and QGP phases. More precisely, we

suggest to identify the bound states of elementary quanta of the Ising model with the diquarks of the SC phase. This identification is supported by the similar behavior of the two binding energies which both depend exponentially on the inverse coupling. Moreover both in the Ising and in the QCD case the two phases have the same global symmetry and the only order parameter which allows to distinguish between them is the binding energy.

Let us stress however that at this stage this identification (as the one which involves the Kertész line) is only a conjecture and is by no means implied by universality. Thus it would be very interesting to study it in other models belonging to the Ising universality class, which could better represent the QCD phase diagram, like the 3d Potts model in a magnetic field considered in [10].

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